

CALCULATORS, MOBILE PHONES AND PAGERS ARE NOT ALLOWED.
Answer all of the following questions. Each (Sub)question is worth 5 points.

1. Use logarithmic differentiation to find $\frac{dy}{dx}$, if

$$y = \frac{(3^{\sec^{-1} x})^3 \sqrt{1+x^3}}{(1 + \cos^{-1} x) \cosh x}.$$

2. Evaluate $\lim_{x \rightarrow \infty} e^x \ln(1 + \sin(e^{-x}))$.

3. Evaluate the following integrals

(a) $\int \frac{x^5}{\sqrt{x^3 + 1}} dx.$

(b) $\int \frac{(\sin 2x) \sqrt{\cos x}}{\csc^2 x} dx.$

(c) $\int \frac{\operatorname{sech}^2 x}{5 + 4 \tanh x - \tanh^2 x} dx.$

4. Determine if the integral $\int_3^{\infty} \frac{1}{\sqrt{x}(9+x)} dx$ is convergent or divergent, and if convergent find its value.

5. Find the length of the curve given parametrically by

$$x(t) = e^t, \quad y(t) = t, \quad 0 \leq t \leq \ln \sqrt{3}.$$

6. Find the area of the region that lies inside the cardioid $r = 4(1 + \sin \theta)$ and outside the circle $r = 4 \sin \theta$.

7. Let \mathbf{a} and \mathbf{b} be two vectors in V_3 . Show that $\mathbf{a} \times \mathbf{b}$ is orthogonal to \mathbf{a} .

8. Let $A(0, 2, 1)$, $B(2, 1, 2)$ and $C(1, 1, 3)$ be points in \mathbb{R}^3 . Find the parametric equation of the line passing through A and B . Find the equation of the plane determined by A , B and C .